### Box 15.1: Examples of the Investigative Approach to Measurement Instruction

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<th>Vignette</th>
<th>Comments</th>
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<td><strong>Example 1.</strong> Mr. Burley suggested that as a service project, his class could seed the patch of lawn that had recently been dug up to repair water pipes. He pointed out that covering the seeded area with plastic would facilitate the growth of grass. To determine how much plastic would be needed, a team was assigned to measure the length and width of the rectangular plot of ground. Mr. Burley included Ariel and Chad on the team because testing had indicated they were unsure of measurement concepts and skills. The lesson was based on a real project. In addition to providing a service to the school community and learning gardening skills, the task was mathematically rich. This complex task provided an opportunity to inquire into new topics and review old topics in a purposeful manner. Note that this lesson reviewed measurement concepts and skills by <em>actually</em> involving children—particularly those who need the review most—in measuring things.</td>
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<td>The measurement team reported back that the plot was 21 feet, 7 inches long by 7 feet, 4 inches wide (rounded off to the nearest inch). The teacher used this opportunity to review rounding procedures.</td>
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<td>The groups set about trying to determine the area. Based on their knowledge of area, [ \text{Alex's group devised the procedure to the right.} ] Mr. Burley then asked the groups to share their solutions. After Alex presented his group's solution, children in several other groups noted their disagreement. &quot;We're not sure exactly what the answer is,&quot; noted Alysen, &quot;but we think it has to be more than Alex's answer.&quot; The teacher withheld judgment and encouraged the children to share ideas and judge for themselves what was reasonable. &quot;Class, Alysen’s group disagrees,&quot; commented Mr. Burley enthusiastically. &quot;Let’s explore this. Alysen, why do you think the answer has to be more than 147 square feet and 28 square inches?&quot; The teacher welcomed the conflict because this can motivate students to explore ideas. He asked Alysen to justify her conjecture.</td>
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| \[ \begin{array}{c}
21' 7'' \\
\times 7' 4'' \\
\hline
147' 28''
\end{array} \] | |
Alysen drew the diagram to the right on the board. "This area," she noted [pointing to part A], "is 147 square feet, and this part [pointing to part D] is 28 square inches. We haven’t even figured out what the other two parts are yet."

"Yah, I can see that," conceded Alex graciously.

"But," continued Alysen, "we don’t know how to multiply feet and inches" (e.g., 21’ x 4”).

Shari suggested, "Why not change the inches to decimals?" She proceeded to write on the chalkboard: 21’ 7” = 21.7’ and 7’ 4” = 7.4’. The class enthusiastically embraced Shari’s suggestions.

"How would you change 7’ 11” into a decimal?" asked Mr. Burley. Shari responded by quickly writing 7.11 on the board and then paused to consider what she had written.

Drawing on his knowledge of decimals, Chico offered, "I don’t think that’s right?"

"No, it can’t be," confirmed Shari, 7’ 11” is more than 7’4” and 7.11 is not more than 7.4.”

"Asked why, Shari added, "Because 7.11 is seven wholes, one tenth and one hundredth and 7.4 is seven wholes and four tenths, which is more."

"How can we change 7’ 4” into a decimal then?" asked Mr. Burley. When there were no responses after several minutes, he prompted: "What’s our whole?"

"Feet," responded Ariel.

"And what part of our whole is four inches?" prodded Mr. Burley.

Looking at her ruler, Ariel offered, "Oh, 4/12?"

Chico quickly agreed, "That’s right, because there’s 12 inches in a foot."
"How can we change $\frac{4}{12}$ into a decimal?" asked Mr. Burley. Marsha noted that $\frac{4}{12}$ was equal to $\frac{1}{3}$ and $\frac{1}{3}$ was equal to the decimal $0.33\overline{3}$. After the class agreed that $7\"$ was $\frac{7}{12}$, Mr. Burley asked how $\frac{7}{12}$ could be changed into a decimal. He then allowed the students to work on this task without further comment.

Exploiting the fraction-decimal equivalents he knew, Alexi offered the following line of reasoning:

$$
\begin{align*}
3\" &= \frac{1}{4} of 12 = 0.25 \\
+ 4\" &= \frac{1}{3} of 12 = 0.33\overline{3} \\
7\" &= 0.58\overline{3}
\end{align*}
$$

So $21' 7" = 21.58\overline{3}\"$

After congratulating Alexi, Mr. Burley asked if there were other ways to change a fraction into a decimal. One student recalled that a calculator could be used by couldn’t remember the procedure. Mr. Burley encouraged the class to check their textbook for the procedure. After the class had rediscovered that a fraction can be converted into a decimal by dividing the top digit by the bottom one, Mr. Burley asked them why dividing made sense. He further prompted them to consider what meanings fractions could have.

Alicia recalled that fractions could have a division, as well as a part-of-a-whole, meaning—and that’s why you divide.

Example 2. McClain, Cobb, and Gravemeijer (1997) reported on a primary-level class that adopted Unifix cubes as a unit of length. After carrying around bags of the cubes to complete various measurement activities, students suggested using a ten-bar (10 cubes pushed together) instead. The problem led to a review of fraction-decimal equivalents. Although the teacher had hoped that the class would recall that fractions can be changed into decimals by dividing the numerator by the denominator, he did not insist they use this procedure.

The textbook was used as a resource.

The teacher wanted his students to understand the rationale for the procedure. He reasoned that by connecting the how to the why, his students might remember the procedure better.

A common fraction such as $\frac{2}{3}$ can represent two things divided among three groups, as well as two parts of a whole divided into three parts.

The class used an informal measure ("Smurf cans," which were represented by Unifix cubes). In the natural course of using this informal measure, children invented their own, more convenient measuring tool (the ten-bar ruler). Use of this tool provided practice skip counting by ten.
During the course of one activity, children had to determine a length 23 units long. Jordana noted that it would be less than two ten-bars long (see Figure A). Sarah suggested that it would be more than two ten-bars long (see Figure B). Jordana argued that Figure B represented 33, not 23, because the three was in the "thirties bar." Sarah countered that the last cube in the first ten-bar was ten, the last cube in the second ten-bar was twenty and the third cube of the third ten-bar was 23.

During the course of a real measurement activity, the issue of how to use a ruler to measure is raised. The teacher allowed the children to debate their conflicting views. The children successfully resolve the debate themselves.

Box 15.1 continued

This explanation satisfied Jordana and the rest of the class.

In the course of measuring and comparing heights, a group had to determine the difference of 67 and 72 cubes. Lynn offered "six" as an answer. Anticipating Lynn’s error, Shana offered "Did you count from 67 and not the 67?"