Fostering Children's Mathematical Power:

An Investigative Approach to K-8 Mathematics Instruction

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PREFACE

Dear Reader:

As a teacher, you will have the awesome responsibility of helping all of your students construct the disposition and knowledge needed to live successfully in a complex and rapidly changing world. To meet the challenges of the twenty-first century, students will especially need mathematical power: a positive disposition toward mathematics (e.g., curiosity and self confidence), facility with the processes of mathematical inquiry (e.g., problem solving, reasoning, and communicating), and well-connected mathematical knowledge (an understanding of mathematical concepts, procedures, and formulas). Imagine having the power to foster children’s mathematical power—the ability to excite them about mathematics, to help them see that it makes sense, and to enable them to harness its might for solving everyday and extraordinary problems. The aim of this teaching guide is to help you achieve this power so that your students can achieve the mathematical power they will need.

Your journey toward empowerment may require you to rethink what and how mathematics should be taught. Unfortunately, the mathematics curriculum, the instructional practices, and assessment methods that you might be accustomed to (e.g., a focus on written arithmetic procedures, lectures and work sheets, and timed tests) are generally not effective in promoting children’s mathematical power. "Improving mathematics education is not a matter of adding a little spice to a dull subject or of making a few minor changes in content or approach. It requires no less than a redefinition of . . . mathematics [instruction] and an understanding that [its] goal . . . must be the development of mathematical power in all students" (Parker, 1993, p. xi). To further these aims, the National Council of Teachers of Mathematics, or NCTM for short, has issued the Curriculum and Evaluation Standards for School Mathematics (Curriculum Standards) in 1989, the Professional Standards for School Mathematics (Professional Standards) in 1991, and the Assessment Standards for School Mathematics (Assessment Standards) in 1995. These Standards do not recommend a band-aid approach to reform (i.e., supplementing existing practices with some new techniques to better accomplish the same old goals)—an approach that has characterized some efforts to improve mathematics instruction in the past. Instead, the Standards propose radical surgery—profound changes in what and how mathematics is taught and assessed.

The heart of the NCTM’s proposed reforms is a new way of teaching mathematics, which we will call the investigative approach. Past efforts to reform mathematics instruction have focused on only one or two of the following aims: making mathematics instruction (a) relevant to the everyday life of students, (b) process-based (e.g., problem-solving or reasoning based), or (c) understandable. The investigative approach attempts to foster mathematical power by accomplishing all three of these aims. By teaching content in a purposeful context, an inquiry-based fashion, and a meaningful manner, this approach promotes children’s mathematical learning in an interesting, thought-provoking, and comprehensible way. This teaching guide is designed to help you appreciate the need for the investigative approach and to provide practical advice on how you can make this approach really happen in your classroom.

Your journey toward empowerment will require you to learn more about mathematics, children’s mathematical learning, and mathematics teaching. As in any domain, powerful knowledge is actively constructed, not passively absorbed by a learner. So we designed this guide not only to dispense a wealth of information but to serve as a catalyst for exploring, conjecturing about, dis...

* For example, proponents of the Social Utility Movement proposed to eliminate from the arithmetic curriculum any subject matter that had no “life value”—that had “little direct use in the life of the child” (Caldwell & Courtis, 1925, p. 68). Proponents of Progressive Education went even further by advocating incidental learning—learning mathematics naturally through personal experience (cf. John Dewey, 1963). William Brownell (1935) agreed that mathematics instruction should not be taught in isolation or without purpose but felt that incidental learning did not adequately develop the meaningful concepts and thinking processes needed for real arithmetical ability. Unfortunately, Brownell’s Meaningful Approach focused primarily on making arithmetic instruction understandable and did little to promote mathematical inquiry. In the 1960s, advocates of the New Math suggested making mathematics understandable by helping children reason out mathematical conclusions. However, this reform movement did not take into account children’s interests, needs, or thinking and, unhappily, often promoted the rote memorization it was designed to avoid (Kline, 1974).
cussing, and contemplating the teaching and learning of mathematics.

To help you actively construct the knowledge necessary to foster children’s mathematical power, this teaching guide has a unique format, one that requires you to interact with the text material. To prompt wonder, speculation, debate, and reflection—the keys for advancing understanding—we have included reader inquiries. These inquiries encourage readers to explore key ideas about mathematics, children’s mathematical learning, and mathematics teaching in a way that models the investigative approach to elementary mathematics instruction recommended by the NCTM (1989, 1991). Thus, we not only discuss how to teach problem solving, but invite you to enhance your own problem-solving ability by solving genuine problems. We not only discuss how to promote understanding, but encourage you to deepen your own understanding of mathematics by employing recommended procedures (e.g., using manipulatives, playing math games, or engaging in discovery-learning activities). We not only discuss teaching philosophies, practices, and methods but prompt you to critically examine your own philosophy, to try out teaching techniques yourself, and to critically analyze curricula materials. We not only suggest integrating technology into mathematics instruction but ask you to undertake activities that involve calculators or computers. Reader inquiries, then, are not a secondary feature of this guide but its core. Our hope is that they will help you become a more reflective and capable mathematical and pedagogical problem solver and, thus, a more effective mathematics teacher than you might have been if you simply read a textbook. Reader inquiries can also serve to illustrate the investigative approach and provide a basis for implementing it in your own classroom.

The process of constructing a deeper understanding of mathematics, how children learn it, and how it can be taught effectively can be greatly aided by discussing these topics with others. Particularly as you tackle the reader inquiries, we encourage you to collaborate with a small group of your peers. This should be helpful in answering the questions or the problems posed and in constructing an understanding of the guide’s content. It will also give you a better understanding of the value of—and the difficulties with—small-group cooperative learning.

Your journey toward empowerment will be ongoing. To accommodate this fact, this guide has another unique feature. Its pages are prepunched to fit a three-ring binder. This feature will allow you to add additional material provided by your instructor, as well as that obtained from sources (e.g., colleagues, books for teachers, workshops, conferences). Moreover, this format permits you to organize and reorganize the material in any way you wish. Some readers may want, for example, to select certain material and fashion it into a plan for a lesson, a unit, or even their own teacher-made curriculum. The notebook format will also enable you to bring only what you need to class or school as opposed to lugging around a weighty reference book. If you tutor a child, it will allow you to pull out a reader inquiry to serve as a basis for a lesson.

To help you get started on your journey to becoming an effective mathematics teacher, we first discuss further the purposes and features of this teaching guide and then explore the critical role a student’s frame of mind plays in the teaching-learning process (chapter 0). In chapters 1, 2, and 3, we provide a rationale and general framework for the rest of the guide. In chapters 4 to 16, we explore mathematical content areas now recommended for K-8 instruction by the NCTM. In chapter 17, we close the guide with some thoughts on organizing instruction and teacher professional development.

It took 8 years to write, rewrite, revise, and re-revise this guide. This process was guided by extensive field testing. We regularly tried out teaching tips and classroom activities with elementary classes or small groups of elementary-age children. Throughout the writing phase, we piloted the reader inquiries and other aspects of this guide with preservice teachers in elementary mathematics method courses.

Nevertheless, as teaching is an ongoing building process, we would greatly appreciate your suggestions for making this learning tool more informative, more readable, and easier to use. You can communicate your ideas in writing to Art Baroody, College of Education, University of Illinois, Champaign, IL 61820, by phone (217-333-4791), by FAX (217-244-4572), or by e-mail (baroody@uiuc.edu).

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