TEACHING TIPS

AIMS AND SUGGESTIONS

Chapter 0 to 3 of the *Student Guide* focused on general issues and framework regarding the teaching of elementary mathematics; chapters 4 to 16 illustrated their applications to specific content areas. The last chapter of the *Student Guide* returns to broad issues and ties together elements introduced or alluded to in earlier chapters. Some instructors will find the contents of this last chapter a fitting way to summarize and end their course.

Other instructors may wish to include all or part of the chapter 17 material in their course introduction (along with chapters 0 to 3) to provide a broader and more complete framework for subsequent chapters. Subunit 17•1•1 (Planning and Conducting Mathematics Instruction) could easily be integrated with Unit 1•2, in which the investigative approach is described. Subunits 17•1•2 (Promoting a Mathematical Community) could easily fit with a discussion of Unit 1•2 or Unit 2•3, both of which include a discussion of promoting mathematical discourse. Subunits 17•1•3 (Meeting Individual Needs) and 17•1•4 (Empowering All Children) could be used to supplement Unit 3•1, in which the issue of readiness and individual differences is first raised. Subunit 17•1•5 (The Uses of Instructional Technology) could be used to extend the brief discussion about using calculators and computers found in Unit 1•2. Unit 17•2 (Professional Development), which provides general advice and sets reasonable expectations about implementing the investigative approach, could be introduced after Unit 1•2 (a discussion of the investigative approach). Unit 17•2 and the final vignette in the Epilogue could be introduced immediately after Unit 0•2, which includes a discussion of affective issues and math anxiety. Some students may find it comforting that Miss Brill, the protagonist in the vignette, not only survives her first year of teaching but prospers, despite incomplete mathematical and pedagogical knowledge, inexperience with teaching, and even errors. The final vignette is an effort to view professional development as a long-term project and may help some students put their course work in perspective.

Yet other instructors may wish to introduce and revisit the issues raised in chapter 17 throughout their course or when students raise questions about them. For example, if a student raises a question about teaching children with learning difficulties, the class can be referred to Subunit 17•1•4 and the Empowering All Children subsection of Some Instructional Resources (on page 17-26 of the *Student Guide*).

Although reading about instructional organization and professional development can be helpful, actually involving adult students in activities and projects that deal with these issues will probably have a deeper impact. For this reason, this chapter will focus on suggested activities and reflective writing activities. Note that the vast majority of suggested activities will require a commitment of considerable time and effort. Thus, students should begin them during the semester, if not at the start of it. Note also that some of the suggested activities, particularly several involving technology, can be integrated with those of earlier chapters.

FOR FURTHER EXPLORATION

QUESTIONS TO CONSIDER

1. Subunit 17•1•4 includes a discussion of the role of teacher expectations and the mechanisms that lead to self-fulfilling prophecies
(see, e.g., Figure 17.4 on page 17-12). Should a teacher look at a child’s academic file before creating an impression of the child?

2. Figure 17.1 below summarizes the phases of teacher professional development. (a) Analyze Miss Brill’s behavior described in the Student Guide in terms of this three-phase model. In the early chapters (chapters 0 and 1, in particular) what was the focus of her concern and at what phase of professional development was she operating? Justify your answers with examples. Did Miss Brill’s focus of concern change during the course of her first year? Justify with examples. Indicate to what extent Miss Brill fit a phase of development. That is, to what extent did her behavior at any given time clearly and unequivocally indicate a particular phase of development? (b) Consider the best and worst mathematics teachers you have ever had. What was the focus of concern of each? Illustrate your assessment. (c) Over the course of a month, visit a teacher’s classroom. Analyze his or her predominant focus of concern and level of professional development. Illustrate your assessment. (d) Use the model of teacher professional development to do a self-assessment. What is your primary focus of concern?

3. Consider each of the following teacher comments. Indicate what level of teacher development each suggests. Briefly justify your answers.

   a. "Are you questioning my judgment young man?"
   b. "What’s the matter Janet? Come on, tell me. You can trust me, can’t you?"

<table>
<thead>
<tr>
<th>Focus of Concern</th>
<th>Sample Questions</th>
<th>Example Comment #1</th>
<th>Example Comment #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>III. Students</td>
<td>Are the students learning? How can their learning environment be improved? Will this problem be too difficult for them?</td>
<td>Most failed the test. What was it that the pupils did not understand and why? Was it the test, material or approach that was inappropriate?</td>
<td>A crowded classroom minimizes learning opportunities and individualized instruction. It’s not fair to my students.</td>
</tr>
<tr>
<td>II. Job</td>
<td>How is my lesson going? Am I speaking clearly, providing enough examples, and giving sufficient feedback? Will we be able to make it through the curriculum in the time left?</td>
<td>Most failed the test. We’ll have to redo that section. I’ll have to make sure they listen to me more carefully this time. I hope this doesn’t put us too far behind schedule.</td>
<td>How do they expect us to do our job properly with overcrowded classrooms? It’s not fair to us as professionals.</td>
</tr>
<tr>
<td>mechanics of teaching</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I. Self</td>
<td>Will the students, other teachers, and principal like me? Will they think I am a good teacher? Will my students respect and obey me? Can I keep the class under control and survive the day?</td>
<td>Most failed the test. My students are going to hate me. Maybe I’m not meant to be a teacher.</td>
<td>I’m not appreciated. Look at these crowded conditions. How does anyone expect me to cope with this?</td>
</tr>
</tbody>
</table>
c. "Andre you look puzzled by my last comment, what's your question?"

d. "The principal thought my lesson was well taught but that I need to keep my class under control better. I think I'll read that book on discipline she suggested.

e. "Teaching is not a profession. It's murder in slow motion; death in daily installments."

f. "The Math Workshop should be good. The guy presents a whole bag of interesting ideas. His lesson ideas are practical and easy to prepare."

SUGGESTED ACTIVITIES

1. (a) Design an opinion survey about using textbooks, homework, or heterogeneous versus homogeneous grouping. (b) Administer the survey to at least 20 in-service or pre-service teachers. Analyze your data in terms of the discussion of the issue in chapter 17 of the Student Guide and record your conclusions. (c) Share your findings with your class, using graphs and statistics as appropriate.

2. (a) Analyze a textbook or curriculum for a grade level of your choice. Overall, what approach to mathematics instruction does it take: the skills, conceptual, investigative, or problem-solving approach? Note its strengths and weaknesses. Indicate how you would use it or what aspects you might use. Indicate how it might be used by a teacher using the investigative approach. (b) Share your report with your class.

3. (a) Collect problems, project ideas, and activities that can serve as worthwhile tasks. Ideally, these items would cut across mathematical topics and/or involve other content areas. (b) Catalogue your collection of worthwhile tasks. (c) Using your collection of worthwhile tasks and the suggestions for teacher-designed instruction described on pages 17-4 and 17-5 of the Student Guide, develop a mathematics curriculum for a quarter.

4. Create a catalog of replacement units. In addition to the name of the unit, reference for each entry: grade level, mathematical topics or content, other subject-matter content, processes of mathematical inquiry. Specify where the unit would fit in a "typical" curriculum (that is, what unit it would replace). Develop an easy-to-use index.

5. (a) Create a menu for a topic or topics and a grade level of your choice. Create or find at least six worthwhile tasks. Sources of ideas might include: the Student Guide, other elementary mathematics textbooks, NCTM Yearbooks, other NCTM publications, teacher-resource or activity books, NCTM journals (particularly Teaching Children Mathematics and Mathematics Teaching in the Middle School, (b) Develop the student instruction sheet for each choice. (c) Collect the material needed for each choice. (d) Field test the menu and revise it. (e) Share your menu ideas with your class.

6. (a) Collect ideas for a mathematics learning station. Record them on 3 x 5 cards or in a notebook. (b) Design one or more station activities, complete with written or audiotaped instructions and the materials needed. (c) Present your ideas to your class.

7. Visit at least five different classrooms during mathematics instruction on at least three different occasions. Analyze each visit in terms of the models of classroom instruction discussed on page 17-5 and 17-6 of the Student Guide and the classroom arrangement (see page 17-7 of the Student Guide).

8. (a) Visit two different classrooms during mathematics instruction on at least three different occasions. Videotape or audiotape the sessions, if possible, and keep notes on the classroom interaction. More specifically, analyze the effectiveness with which a teacher asks questions and responds to students' answers (see guidelines on pages 17-7 to 17-9 of the Student Guide). Indicate a teacher's strengths or how he or she could improve. Did the teacher use questions that promote mathematical power (see Appendix H on the inside back cover of the Student Guide). If so with what frequency: almost always, usually, sometimes, infrequently, or never? How did the children respond to these questions? (b) Present a summary of your findings to our class. (c) Videotape at least three of your
own mathematical lessons. Analyze your performance as outlined above.

9. (a) Choose a mathematical topic and devise three questions that require students to explain an answer, to analyze a situation, to apply learned material, to integrate ideas, or to evaluate whether a position makes sense. (b) Try out these questions with a class of elementary students. Evaluate the effectiveness of the questions and how they could be modified to be even more effective. (c) Present your results to your class.

10. Consider Box 17.2 on page 17-7 of the Student Guide, and then devise a Home Connections message on five different topics.

11. (a) Canvass practicing teachers in person or on the world wide web for their advice about obtaining parent support. (b) Summarize, categorize, and evaluate your findings in a report. Indicate which ideas you think might be particularly helpful. (c) Try implementing at least two suggestions for obtaining parent support described on pages 17-9 and 17-10 of the Student Guide. (d) Evaluate the success of your efforts. (e) Present your findings to your class.

12. (a) Develop a plan for round-robin learning stations for a one- or two-day lesson on a topic and grade level of your choice (see page 17-11 of the Student Guide). Specify the content and process aims for each station, what material would be needed, the instructions for the children, and the procedure for the station. (b) Construct the stations for the round-robin lessons, including the materials and instructions. (c) With at least one station, use technology such as calculators, a computer, overhead projector, videotapes, audiotapes, or CDs. (c) Try out your round-robin lessons and evaluate its success. (d) Report your efforts to your class.

13. (a) Observe a classroom or a videotape of a classroom. Using Figure 17.4 (page 17-12 of the Student Guide), analyze the teacher's behavior. What expectations does it communicate? Are the expectations for boys and girls, children of different ability levels, and children of different races or social classes similar or obviously different? (b) Share your findings with your class.

14. (a) Interview at least one child facing special challenges in learning mathematics for about 20 minutes on each of at least four occasions. (b) Write up a case study of the child (children), indicating the principle sources of difficulty, how the school is attempting to help, and your recommendations for helping the child. Relate your discussion to the points made in Subunit 17•1•4 (pages 17-11 to 17-16 of the Student Guide). (c) Report your case study to your class.

15. (a) Evaluate at least three examples of instructional software. Indicate whether each involves using the computer as a trainer, tutor, tutee, or tool (see pages 17-18 and 17-19 of the Student Guide). Analyze each in terms of the constructivist applications of technology summarized in Box 17.8 on pages 17-20 and 17-21 of the Student Guide. Consider whether or not a program has the following characteristics:

- The menu uses icons so that the program or subprograms can be easily accessed without the need for remembering or keying in their names or codes.

- The instructions are self-explanatory.

- The program actively involves students—requires the student to think and act—rather than watch passively.

- A help menu is available to answer questions about the program.

- Students have the option of ending the program or restarting it at any time.

(b) Try out the software with children of an appropriate age. Evaluate the their reaction to the software and how they benefited educationally. (c) Demonstrate the software to your class and share your report with it.

16. (a) Visit at least three elementary classrooms. Describe the technology used in each. Evaluate how effectively the technology was being utilized. Indicate what additional technology or changes in its use you recommend. (b) Share your report with your class. (c) Use appropriate technology to make your observations, evaluation, and presentation.
17. Use a word processor to complete (a) a report on at least one other suggested activity, or project, (b) your group's writing assignments, or (c) your individual journal.

18. Use e-mail to communicate with your group about completing (a) homework assignments, (b) a suggested activity, or (c) a project. Document your communications either by printing out hard copies or copying to your instructor.

19. (a) Use e-mail or a web board such as TBase to communicate with instructor about questions you have regarding the course, readings, assignments. (b) E-mail your group's or your individual journal responses to your instructor.

20. (a) Use e-mail to communicate with your supervising teachers about expectations, schedules, lesson planning, and so forth. (b) Use e-mail to communicate with a practicing teacher or other mentor about questions you have about teaching.

21. Use a conferencing system such as WebBoard or First Class to discuss with your group (a) homework assignments, (b) suggested activities, (c) projects, or (d) required readings.

22. Create a personal homepage and print it out.

23. Develop (a) a multimedia professional portfolio including videotape of at least two exemplary mathematics lessons or (b) a professional e-portfolio.

24. (a) Search the world wide web for web sites regarding mathematics instruction appropriate for a grade level of your choice. Search engines you might find helpful include Alta Vista, ElNet Galaxy, Infoseek, Lycos, WebCrawler, and Yahoo. You can find these search engines at, for example, the following web sites: http://home.netscape.com/escapes/search/ntschrmd-1.html or http://www.indiana.edu/world.html. (b) Construct a WWW Directory of the relevant web sites. Include in the directory a table of contents, a printout of the homepage of each entry with its URL, a brief description of the information and resources available at the web site, a brief description of how teachers could use the information or the web site in their instruction, and a brief evaluation of the web site. (c) Reproduce your directory for your class and provide it with an overview of the directory. (d) Link appropriate web sites to your professional e-portfolio.

25. In conjunction with another suggested activity in this or previous chapters, develop a lesson plan or project that requires creating a simple database and using a spreadsheet.

26. (a) Develop a multimedia lesson on a mathematical topic and grade level of your choice. (b) Present the lesson to an elementary class. (c) Evaluate your lesson, particularly your use of multimedia technology. (d) Present your lesson idea and evaluation to your class. Use appropriate technology to do so.

27. (a) Explore the world wide web for professional listservers Make a list of their names, electronic address, and key information. (b) Join at least one professional listserv.

28. The Federal government, universities, and private organizations have set up web sites dedicated to improving instruction in general and mathematics instruction in particular. (a) Visit at least one of the web sites listed below or on pages 17-27 and 17-28 of the Student Guide. (b) Write a report on the web site. Include in your report what information can be found at the web site, the last time it was updated, and how valuable the web site might be to a new elementary teacher. (c) Share your report with your class.

- "Computers and Classrooms: The Status of Technology in U.S. Schools" summarizes the results of a computer-use survey done by the Educational Testing Service (ETS).
- "Promising Practices: New Ways to Improve Teacher Quality" is a report released by the U.S. Department of Education (http://www.ed.gov/pubs/PromPractice/index.html or e-mail edinfo@inet.edigov for information).
29. (a) Read the following article: “The Link Between Technology and Authentic Learning” by Barbara Means and Kerry Olson appearing in *Educational Leadership, Volume 51, Number 7* (April 1994), pages 15-18. (b) Write a report that summarizes the key points of the article. Include what the authors say about the impact of technological advances in the 1960s, 1970s, and 1980s, the results of a project funded by the Office of Educational Research and Improvement, the positive effects of using technology in classrooms, and teachers’ role in a technology environment. Critically evaluate the authors’ conclusions. Indicate how the report is consistent or inconsistent with Subunit 17·1·5 (pages 17-16 to 17-22 in the *Student Guide*). (c) Present your report to your class.

30. (a) Request a copy of one or more of the NCTM’s position statements using e-mail (infocentral@nctm.org or FAX-on-Demand service: 800-220-8483) or read the document(s) online at www.nctm.org. Position statements include Calculators and the Education of Youth, Early Childhood Mathematics Education, the “Every Child” statement, Guiding Students regarding Their Mathematics Education, The Mathematics Education of Underrepresented Groups, Mathematics for Second-Language Learners, Mathematics Leaders in Elementary/Middle Schools, Research in Mathematics Education, Teaching Mathematics in the Middle Grades, and The Use of Technology in Learning and Teaching Mathematics. (b) Write a report on what you learned and present it to your class.

31. (a) Visit the NCTM web site (www.nctm.org) and find at least three of the following:
   - updates on, and links to, the latest news and reports on mathematics education;
   - job listings in the Online Classifieds;
   - information on regional and national NCTM conferences;
   - membership information
   - excerpts from NCTM journals and the *News Bulletin*, and
   - NCTM Educational Materials and Products Catalog.

(b) Write a report on what you found and share it with your class.

32. (a) Search the following websites or others for instructional activities or worthwhile tasks: Do Math Web Site maintained by the NCTM (www.domath.org), The MathMagic K-12 project (http://forum.swarthmore.edu/mathmagic/index.html), MegaMath Project (http://www.c3.lanl.gov/mega-math/index.html), McGraw Hill (http://www.mmhschool.com), Scott Foresman Addison Wesley (http://www.sf.aw.com), or Silver Burdett Ginn (http://www.sbgmath.com). (b) Write a report on what you found, including a description of the activities or worthwhile tasks you found and how they could be used as the basis for a lesson that uses the investigative approach. Also evaluate the value of the website for elementary teachers who are interested in using the investigative approach or the conceptual approach. (c) Share your report with your class.

33. (a) Develop a lesson plan that creates a real need for computer programming (see, e.g., Box 17.7 on page 17-20 of the *Student Guide*). (b) Devise an appropriate program using Chipmunk Basic or a programming language of your choice.

34. Obtain a Link2Learn Professional Development CD by contacting the Regional Office of Education Professional Development Services via the internet (http://www.dupage.k12.ib.us), in writing (1 South 331 Grace Street, Lombard, IL 60148), or by phone (630-495-6080). (a) Write a report on the CD that includes an evaluation of the Technology Tutorials, Featured Teacher Case Studies, Classroom Activities, and Selected Software for Teachers. (b) Share your report with your class.

35. (a) Collect information about one of the following elementary or middle school curriculum projects sponsored by the National Science Foundation (NSF). This can be done by calling the contact person listed, via the internet, or other resources. (b) Write a report about the curriculum including a brief summary of its rationale and aims, targeted grade level, and content and process focus. Also include your evaluation of the curriculum. (c) Share your report with your class.
Elementary Math

University of Chicago School Mathematics Project (UCSMP)
Max Bell (312) 702-1563

Cooperative Mathematics Project
Laurel Roberston
Developmental Studies Center
Oakland, Ca.
(510) 533-0213

Investigations in Number, Data and Space
Susan Jo Russell
TERC
Cambridge, Mass.
(617) 547-0430

TIMS: Teaching Integrated Math and Science
Phil Wagreich
University of Illinois, Chicago
(312) 413-3019

Middle School

The Connected Mathematics Project
Kathy Burgis
Michigan State University
(517) 432-3635

Mathematics in Context: A Connected Curriculum for Grade 5-8
Thomas Romberg
University of Wisconsin
(608) 263-4285

Middle School Mathematics Through Applications Project
Shelley Goldman
The Institute for Research on Learning
Palo Alto, Ca.
(415) 497-7963

Seeing and Thinking Mathematically
Glen Kleinman
Education Development Center (EDC)
Newton, Mass.
(617) 969-7100

Six Through Eight Mathematics (STEM)
Rick Billstein
University of Montana
Missoula
(406) 243-2603

36. (a) Visit www.booksonline.com and examine the books on mathematics, history, puzzles, and theory, such as:

- *The joy of π* by David Blatner
- *Life by the Numbers* by Keith Devlin
- *The Last Recreations: Hydras, Eggs, and Other Mathematical Mystifications* by Martin Gardner
- *The Pleasures of Counting* by T. W. Körner
- *Can You Win? The Real Odds for Casino Gambling, Sports Betting, and Lotteries* by Mike Orkin
- *Fractal Cosmos: The Art of Mathematical Design* by Amber Lois

(b) Order, read, and write a book report on one or more of the books above. Include in your report what you learned about mathematics from a book, how you could use the book or its aspects as the basis for a mathematics lesson, and how a book furthered your professional development.

**HOMEWORK OR ASSESSMENT**

**QUESTIONS TO CHECK UNDERSTANDING**

1. Circle the letter of each statement below that, according to the *Student Guide*, is true.

a. Unlike autocratic teachers that use a direct-instruction approach, more democratic teachers that actively involve students in lessons do not need to plan ahead.

b. Once an inexperienced teacher makes a plan, a good rule of thumb is to not deviate from it.

c. Effectively implementing the investigative approach will probably require teachers to design their own mathematics curriculum.

d. The spiral curriculum is based on the assumption that children only gradually construct a relatively accurate and complete understanding of mathematics.

e. Using typical textbooks as the foundation of instruction is consistent with the con-
ceptual approach but not the investiga-
tive approach.

f. Typically whole-class instruction and small-group work should not be inte-
grated.

g. Regular homework consisting of work-
sheets is necessary for mastering basic
skills.

h. Yes-no questions should be used regularly, because they can be asked often without much loss of time.

i. Teacher questions should focus on factual recall.

j. Research suggests that waiting for stu-
dents to respond to questions undermines their positive disposition, because it makes them nervous.

k. Teachers should avoid homogeneous grouping and rely on heterogeneous group-
ing.

l. To avoid the pitfalls of self-fulfilling prophecies, teachers should encourage students to set their own expectations.

m. Many girls lose interest in mathematics because of cultural expectations.

n. The focus of instruction for gifted children should be acceleration through the cur-
riculum.

o. Elementary-level instruction should avoid the use of calculators.

p. The use of calculators interferes with the mastery of basic number combinations.

q. Using computers should begin when chil-
dren enter the intermediate-level grades.

r. Research suggests that computers are not an adequate substitute for concrete ma-
ipulatives.

s. Programming a computer is using it as a tool.

t. Using a computer to teach material in a logical manner is using it as a trainer.

u. A disadvantage of using computers is that it tends to decrease student-teacher inter-
action.

2. How could Aliya’s rule (page 17-3 in the Student Guide) be adapted for (a) base four, (b) base seven, and (c) base twelve?

3. Ms. Clausen argued that whole-class instruction and discussion is incompatible with the investigative approach. Is her view consistent with that of the Student Guide? Why or why not?

4. Indicate whether each of the following teacher questions is effective or ineffective, according to the Student Guide. Briefly explain why a question is ineffective.

a. Is the probability zero?

b. Can the answer to this probability problem be 1.5? Why or why not?

c. Tell me why you think the game is fair?

d. If there are four different but equal likely choices, then the probability of picking any one item could not be \( \frac{1}{5} \), could it?

e. Look, there are four equally likely outcomes. What is the probability of any one of them happening? Students this is not brain surgery.

f. Does the solution change if the problem reads ”pick a red block and then a blue block from a box of four differently colored blocks” instead of ”pick a red block and a blue block from a box of four differently colored blocks”?

g. Is the answer less than .5, exactly .5, or MORE THAN .5.

WRITING OR JOURNAL ASSIGNMENTS

1. Trafton (1984) has argued that understanding is a unit objective rather than a lesson objec-
tive. In what ways do you agree and disagree with this statement?

2. (a) What have you learned about mathematics this semester? (b) What have you learned about children’s mathematical learning? (c)
What are the three most important thing you have learned about teaching mathematics?

3. Compare your general approach or plan for teaching mathematics now with those you had at the beginning of the semester. In what ways has it remained the same? In what ways has it changed? In what ways are you considering a change?

4. Describe how the planning and general conduct of instruction for the investigative approach is the same or different than that for (a) the skills approach, (b) the conceptual approach, and (c) the problem-solving approach.

5. Write about an experience either as a student or a teacher where a lesson took an unexpected twist. How did the teacher handle the situation? What were the positive outcomes? What were the negative outcomes and how could the teacher have handled the situation more productively? Compare your experience with that described in Box 17.1 on page 17-3 of the Student Guide.


7. What role did teacher expectations play in your own mathematical development? Were teacher expectations generally a positive or negative factor? What sources of expectations do you recall having an impact? In what ways might these sources of expectation have affected your teachers for better or worse? How were your teachers’ expectations reflected in their behavior toward you?

8. Analyze a school’s or your own elementary mathematics instruction in terms of the characteristics of exemplary mathematics programs for women and minorities (see Box 17.5 on page 17-14 of the Student Guide).

9. Write a report about the challenges to learning mathematics confronting a particular group of children. Describe what the group’s special challenges are and why they face them. Summarize recent research about the issue. Indicate what a teacher can do to help this group of children successfully surmount the challenges they face.

10. (a) How do you feel about using calculators and computers in mathematics instruction? (b) Do you agree with the NCTM’s recommendation that calculators should always be available? Justify your response.

11. Develop a case for using (a) calculators and (b) computers in elementary classrooms. Include counterarguments and how you would address them.

12. Write a report on how electronic technology has changed elementary mathematics instruction.

13. Video-conferencing or electronic-based instruction is gaining attention. What are the pros and cons of such long-distance learning methods? Evaluate these methods in terms of the instructional guidelines outlined in Unit 1-2 of the Student Guide.

14. Do a self-assessment (a) of your progress in this course, (b) in terms of the key characteristics of good teachers described on page 17-23 of the Student Guide, and (c) of your readiness to teach elementary mathematics.

15. Complete the following statements: (a) If there was one thing I could change about this course, it would be . . . . (b) If there was one thing I would not change about this course, it would be . . . .

PROBLEMS

- **Tricky Thirds** (3-8)

Ten match sticks are used to create a rectangle as shown below. How could five additional match sticks be used to divide this rectangle into thirds? Exactly five match sticks must be used, and match sticks may not be broken into parts.
**Lights Out (♀ 3-8)**

In one room, there is a three-switch light switch. In an adjoining room are three lamps, each controlled by one of the switches. Connecting the two rooms is an airtight door. Your job is to figure out which switch controls which lamp by making three switch flips. All the switches are currently in the OFF position, as shown in the figure below. So to make three switch flips, you could, for example, flip one switch to ON, then to OFF, then to ON again, or you could flip each of the switches to ON. The door will open when and only when there is a third flip, and then and only then can you enter the second room to see which lamp(s) is (are) on.

![Lights Out Diagram](image)

**Small Change (♀ 3-8)**

Gustov has exactly four coins: a penny, a nickel, a dime, and a quarter. (a) If he haphazardly gave three coins to his sister Sher, what amounts of money could she get? How could you prove that you have listed all possible answers. (b) If he haphazardly gave two coins to his sister Sher, what possible combination of coins could she get? How could you prove that you have all possible answers. (c) For part b, what proportion of the time would she get a larger amount of money than he would? How could you prove it?

**A Mysterious Change (♀ 7-8*)**

A polygon is partitioned into four parts as shown in Figure A below. These four parts are then rearranged to form Figure B. If each of the four parts in Figure B has the same area as its corresponding part in Figure A, why does Figure B have a hole in it? Prove your answer mathematically.

![A Mysterious Change Diagram](image)